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A Methodology of Metamaterial Effective Permittivity and Permeability Value Measurement

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Abstract: Developers of metamaterial-based devices in our dais face a serious problem—they can guess the properties of metamaterial itself only by analyzing the overall parameters of the complete device. As such constructions are periodical metal-dielectric structures, the standard methods of their permittivity and permeability measurement cannot be implemented. In this paper a methodology of metamaterial effective permittivity and permeability determination, based on their complex reflecting and transmitting coefficient measurement, is suggested. The base of the presented algorithm is evaluations, shown here, which determine the dependence of dielectric parameters of the sample under test on its measured S-parameters. For practical measurement some constructions of measuring tools, which allow metamaterial parameter determination in case of using longitudinal electromagnetic wave with vertical and horizontal polarization of electric field and normally incident wave, are suggested. A form of realization and a band, in which measurements can be provided, are presented for each type of measuring tools. An advantage of all constructions is that they are closed structures and do not feel the influence of surroundings. The schema of connecting the measuring tools to a vector network analyzer is detail reviewed. A question of measuring schema element influence on the accuracy of parameter determination is discussed separately. As the constructions of measuring tools do not allow direct measuring device calibration, a method of compensation of their influence on the measured parameters accuracy, based on the mathematical processing of the obtained data, is suggested. Practical results of metamaterial sample effective permittivity and permeability determination, made using the described algorithm and two different measuring tools, are shown. The made experiment has shown a good correlation of the obtained data between each other and with the computed parameters of the sample. So the paper presents the detailed description of the methodology of metamaterial effective permittivity and permeability determination, with the help of measuring their complex reflecting and transmitting coefficient, some measuring tool constructions are suggested, which allow practical measurements, measuring circuit calibration methods are described. The presented experimental data confirm that the suggested algorithm can be used.

Key words: Band gap, effective value, frequency-selective surface, metamaterial, permeability, permittivity.

1. Introduction

In our days metamaterials are more and more applicable for various microwave devices design [1, 2]. They are used as elements of amplifiers [3], filters [4, 5], power couplers [5] and other applications. They are also widely used for antennas [6, 7] and antenna arrays [8] constructing. Their unusual properties, which are conditioned by the possibility of obtaining a negative reflective index value, allow significant improvement of characteristics of microwave devices, where they are implemented.

But the developers of such devices face a serious problem—self parameters of metamaterial can be determined only indirectly by analyzing the characteristics of the complete device. As metamaterials are periodical metal-dielectric structures, which properties are determined not only by the properties of their components, but the same time by their constructions, such methods of their permittivity and permeability determination as method

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of complete resonance [9], method of dielectric resonator [10] and other traditional methods are not applicable.

At now several methods of metamaterial working band measurements are suggested, such as probe method [11] or method of directive antennas [12]. But they provide only a qualitative assessment of the band gap width and a suppression level, provided by the structures, and do not allow evaluating permittivity and permeability values.

The paper presents a method of metameterial effective permittivity and permeability values measurement and construction of measuring tools, which are necessary for measurements in case of different electromagnetic field polarization.

2. Determination of Structure Effective Permittivity and Permeability Values with the Help of Complex Transmission and Reflection Coefficients

Basically metamaterials can be considered as anisotropic environments. Their construction is a periodical metal-dielectric structure, which has at least one symmetry axes. As the size of metamaterial construction resonance elements are usually less than the tenth part of the wave length in the structure operating band, which can be considered as isotropic environment in case of electromagnetic wave propagates along the symmetry axes and has a linear (vertical or horizontal) polarization of electric field.

Let's consider metamaterial as a structure with the length d, with an electromagnetic wave normally falling on its surface (Fig. 1). Vectors E and H are electric and magnetic components of the field, P—pointing vector. Indexes «f», «r» and «tr» marks components of electromagnetic field of falling, reflected and transmitted wave respectively.

Such a structure can be represented as an equivalent S-type quadripole, which complex reflection S_{11} and transmission S_{21} coefficients can be presented in the following way [13, 14]:

$$S_{11} = \frac{R_{01}(1-e^{i2nk_0d})}{1-R_{01}^2 e^{i2nk_0d}} \tag{1}$$

$$S_{21} = \frac{(1-R_{01}^2)e^{i2nk_0d}}{1-R_{01}^2 e^{i2nk_0d}} \tag{2}$$

where,

$$R_{01} = \frac{Z-1}{Z+1} \tag{3}$$

n—slowdown coefficient; R_{01}—multiplication coefficient in Eqs. (1) and (2) [13, 14]; Z—complex wave resistance of equivalent quadripole; $k_0 = 2\pi/\lambda$—wave number; d—structure length.

From Eqs. (1)-(3) for complex wave resistance and slowdown coefficient determination can be obtained [13-16]:

$$Z = \sqrt{\frac{(1+S_{11})^2 - S_{21}^2}{(1-S_{11})^2 - S_{21}^2}} \tag{4}$$

$$n = \frac{1}{k_0 d}\cos^{-1}\left[\frac{1}{2S_{21}}\left(1-S_{11}^2+S_{21}^2\right)\right] + \frac{2\pi m}{k_0 d} \tag{5}$$

where, m—an integer number, equal to a number of

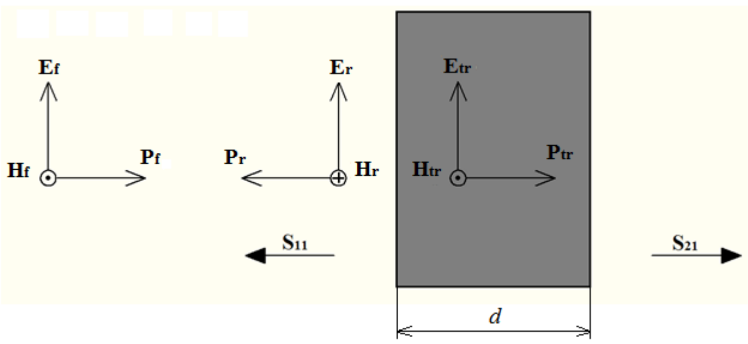


Fig. 1 Electromagnetic wave normally falling on the metamaterial surface.

wave half on the measurement frequency, which can be arranged along the structure length *d* [16].

If we know the metamaterial complex wave resistance and slowdown coefficient, its effective values of permittivity (ε) and permeability (μ) can be determined using the equations:

$$\varepsilon = \frac{n}{Z} \tag{6}$$

$$\mu = nZ \tag{7}$$

So effective values of metamaterial structure permittivity and permeability can be determined by measuring their complex reflection and transmission coefficients.

3. Metamaterial Complex Reflection and Transmission Coefficient Measurement

As generally the metamaterial structure is not symmetrical along all main coordinate axes [2], its properties in different directions and in case of different falling electromagnetic wave polarization cannot be the same. According to the reason it is expedient to make structure measurements for each field polarization separately.

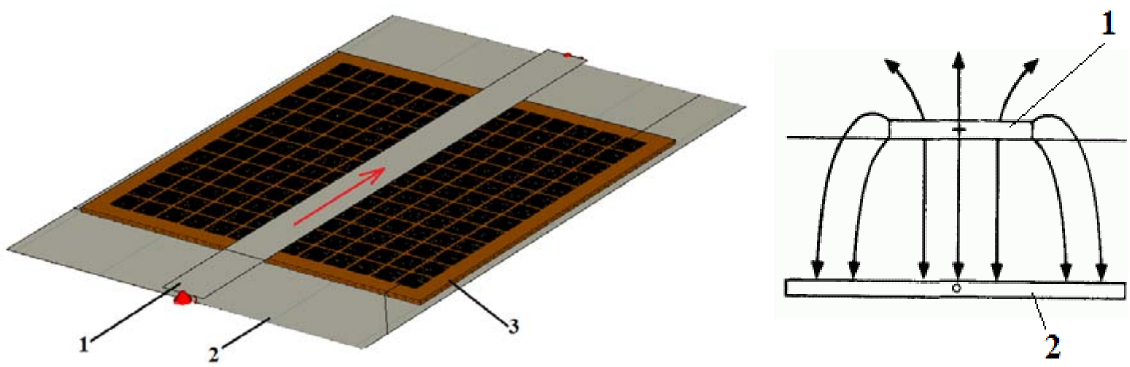
3.1 The Case of Propagating of Electromagnetic Wave with Vertical Polarization of Electric Field along the Metamaterial Working Surface

In this case to determine the complex reflection and transmission coefficient values, the method of measurement with the use of microstrip line can be implemented [17, 18].

The idea of the method is that between microstrip line conductor (1) and its ground plane (2), a thin plate of the material under test (3) is placed (Fig. 2a). The direction of electromagnetic wave propagation is shown in Fig. 2a by an arrow. The electric field in the construction is mostly concentrated between the line conductor and the ground plane and has a vertical polarization in the area [19] (Fig. 2b). The advantage of the method is the possibility of the used microstrip line coupling in a wide frequency band [18], which allows creating a universal measuring tool.

The example of such a construction realization is shown in Fig. 3. A microstrip line conductor with the width *a* and length *b* is placed on the high *c* above the ground plane with the length *b* and width *d*.

A fabricated measuring tool sample (Fig. 3b) is an FR4 PCB with dimensions *b* = 200 mm and *d* = 100 mm fixed on the high *c* = 5 mm above the ground plane of the same size with the help of dielectric stages. On the PCB side facing the ground plane a microstrip line conductor with dimensions *a* = 20 mm and *b* = 200 mm is made. There is no metallization on the opposite side of the PCB. The measuring tool connection to a network analyzer is realized by coaxial cables, which central wires are connected to opposite edges of the microstrip line conductor and their shield is connected to the ground plane. The



(a) (b)

Fig. 2 An overall view of a measuring tool for metamaterial parameters measurement with the help of a microstrip line (a) and its electric field distribution (b).

measured complex reflection and transmission coefficients of the tool (Fig. 3c) show that the construction is coupled in the frequency band 1-2 GHz.

3.2 The Case of Propagating of Electromagnetic Wave with Horizontal Polarization of Electric Field along the Metamaterial Working Surface

To measure the values of complex reflection and transmission coefficients values in this case, a construction, presented in Fig. 4, is suggested. It contains two dielectric substrates (1) placed one against another on the distance *b*, with T-shape conductors (2) with the width *a*, made on their top side. Conducting ground planes (3) with dimensions *c*d* are placed from the top and from the bottom of the substrates on the distance *e* between each other in the way that T-shape conductors appear right in the center between them. The sample under test (4) should be placed on the bottom ground plane between dielectric substrates. Top and bottom ground planes are connected one to another by conducting stages (5).

The electric field distribution in the structure is presented on Fig. 4b and is close to a distribution of the gap line electric field [19]. In the area between two T-shape conductors the electric field polarization can be considered as horizontal. The sample under test should be placed directly in this area.

The manufactured sample of the construction (Fig. 4c) has the following dimensions: *a* = 80 mm, *b* = 150 mm, *c* = 100 mm, *d* = 200 mm, *e* = 5 mm. The dielectric substrate material (1) is FR4. The conducting stages (5) are metal stages for PCB with the high 5 mm. The measuring tool connection to a network analyzer is realized by coaxial cables, which central wires are connected to a horizontal part of the T-shape conductors, and their shield is connected to a bottom ground plane. Complex reflection and transmission coefficients, presented in Fig. 4d, show that the sample is coupled only in a respectively narrow band 1.52-1.58 GHz.

3.3 The Case of Normal Incidence of Electromagnetic Wave on the Metamaterial Surface

In a number of papers [20, 21] results of measuring frequency-selective surface parameters with the help

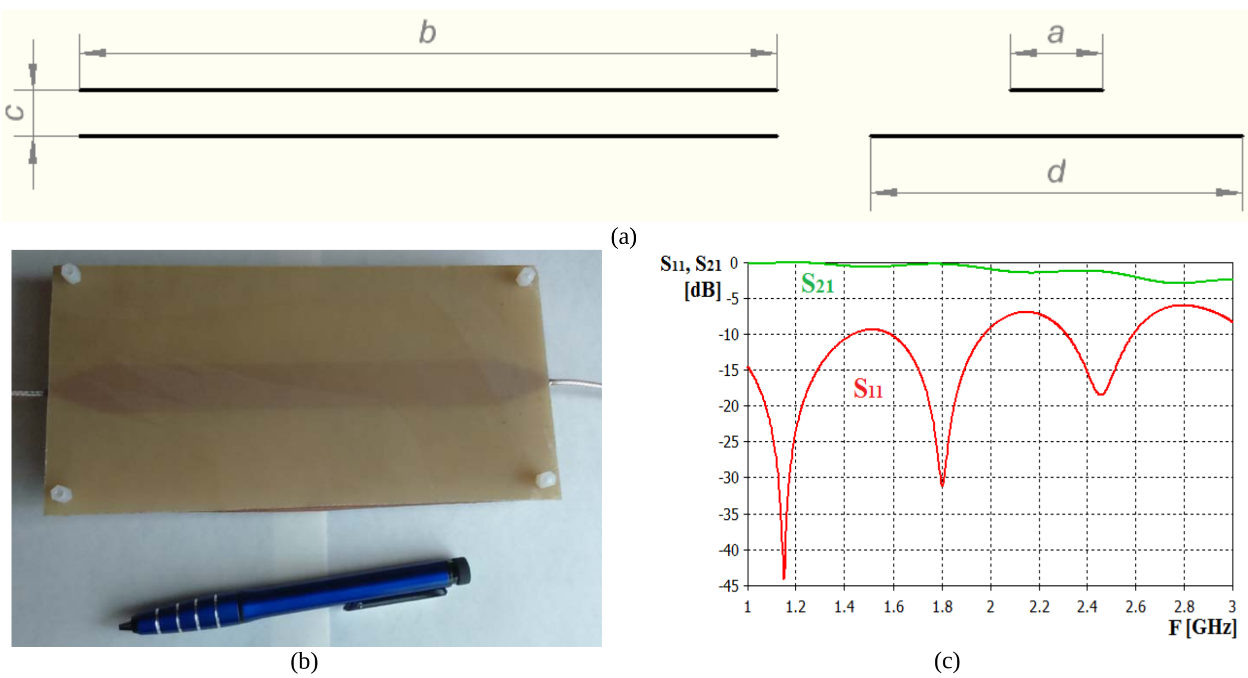


Fig. 3 Construction of the tool for measuring metamaterial parameters with the help of microstrip line (a), its manufactured sample (b) and the dependence of the sample reflection and transmission coefficients on the frequency (c).

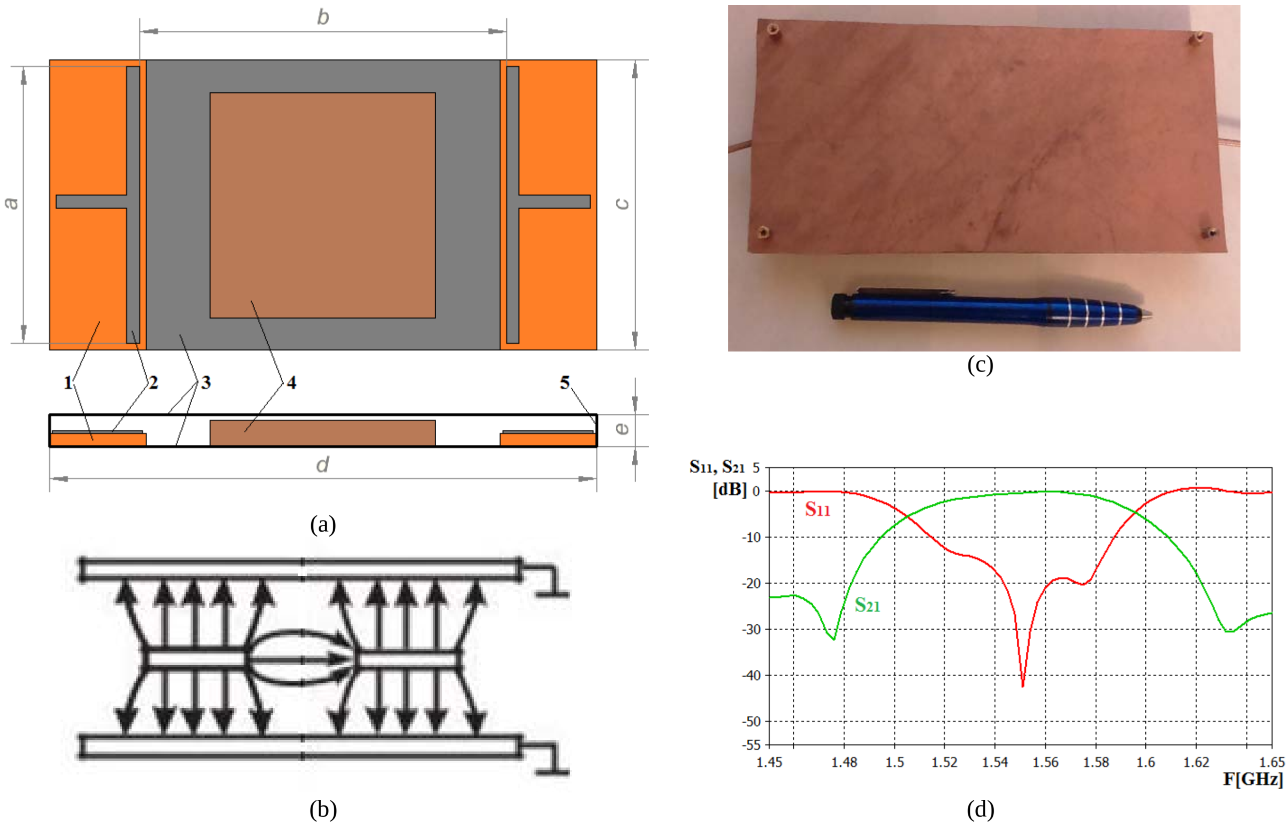


Fig. 4 Construction of the tool for metamaterial parameter measurement in case of horizontally polarized wave propagation along its surface (a), electric field distribution in it (b), manufactured sample (c) and the dependence of reflection and transmission coefficients on frequency (d).

of directive antennas, placed from the both sides of the sample under test, are presented. But such measuring stands have the following disadvantages: they are bulky, difficult in operation as requiring accurate installation in case of replacement or assembling; they are open systems sensitive to surrounding object influence, which means that the testing room should be specially prepared.

Because of these reasons it is better to use closed structures for metamaterial effective permittivity and permeability determination in case of normal incidence of electromagnetic wave on its surface. One of the systems, which enable measuring in a wide frequency band, is presented in Ref. [22] and in Fig. 5a. It consists of dielectric resonators (1) with diameter a, installed on dielectric stages (2) inside a volume, bordered by metal glasses (3) with inner diameter b. A gap with width c is made between glasses, where a sample under test (4) is placed. The resonator (1) feeding is made by probes (they are not shown on the picture).

Fig. 5b presents a result of simulation of reflection and transmission coefficients of the construction with dimensions a = 40 mm, b = 100 mm, c = 5 mm. It can be seen that the structure is coupled in the frequency band 1-2 GHz.

3.4 Measuring Stand Construction and Its Calibration

The measurement of complex reflection and transmission coefficients can be made by using the vector network analyzer. The analyzer software should provide a possibility of uploading the complex values of reflection and transmission coefficients in a convenient form (.txt or .dat files for example) for further calculations using Eqs. (4)-(7). Fig. 6a illustrates the measuring tool connecting schema. Fig. 6b shows a schema of signal distribution in a network analyzer [23]. The properties of distorting

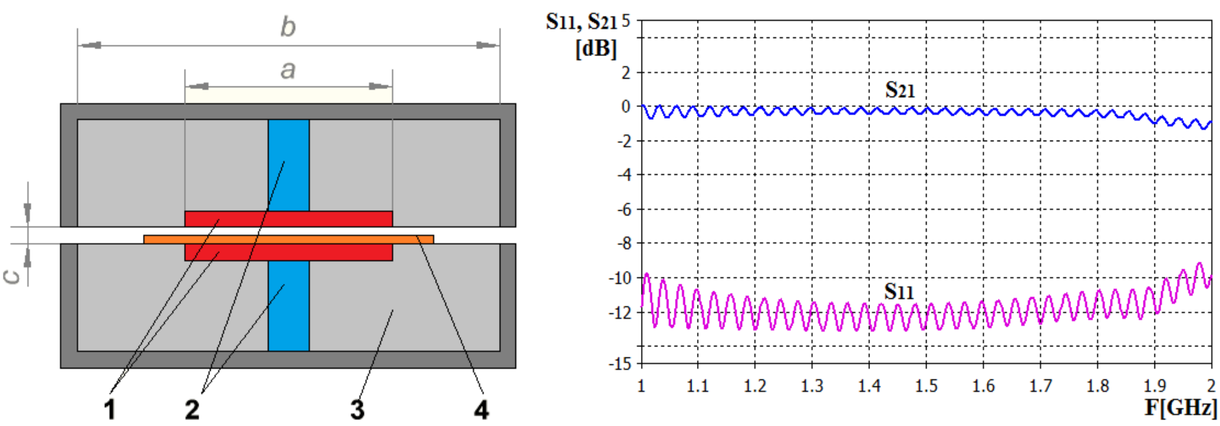


Fig. 5 Construction of a tool for metamaterial parameter measurement in case of normal incidence of electromagnetic wave on its surface (a) and frequency dependence of the tool sample reflection and transmission coefficients (b).

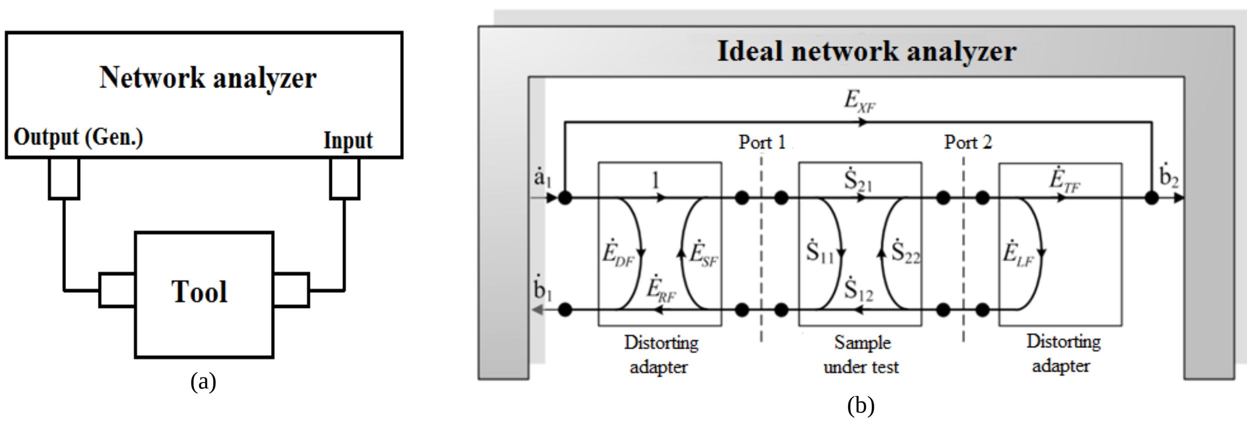


Fig. 6 Schema of connecting a measuring tool to a network analyzer (a) and vector network analyzer model (b).

adapters are determined by their complex reflection and transmission coefficients, which are marked in Fig. 6d as E_D, E_S, E_R, E_T and E_L. Additional index «F» marks the direct signal propagation and «R»—the reverse one. The parasite signal penetration from the generator is marked as E_X.

As it is seen from the Fig. 6b the measuring schema shows not only reflection and transmission coefficients of the structure under test, but also some internal losses. So to obtain self reflection and transmission coefficients of the sample, the measuring stand, assembled according to the schema in Fig. 6a, should be calibrated. The calibration of output connectors and connecting cables should be made according to a user manual of the network analyzed, used for measuring. But the influence of the measuring tool cannot be compensated using the standard calibration procedures.

According to Refs. [23, 24] and as it is shown in Fig. 6b, the measuring tool can be considered as a distorting adapter, serially connected with the sample under test in the schema. So the metamaterial self complex reflection and transmission coefficients can be obtained using equations [24]:

$$S_{11}^{tr} = S_{11}^{m2} - S_{11}^{m1} \tag{8}$$

$$S_{21}^{tr} = S_{21}^{m2} - S_{21}^{m1} \tag{9}$$

where, S_{11}—complex reflection coefficient; S_{21}—complex transmission coefficient; index tr—the true value of the coefficient; index m1—the measured value of the tool without the sample under test inside; index m2—the measured value of the tool with the sample under test inside.

So to obtain self (true) values of metamaterial complex reflection and transmission coefficients the sample and measuring tool should be measured, and calculations according to Eqs. (8) and (9) should be made.

3.5 Practical Measurement of Metamaterial Parameters

A sample of a dual band mushroom-type metamaterial [7, 25-27], which overall view is shown in Fig. 7a, was taken for the experiment. The structure parameters were chosen in the way to provide signal suppression in frequency bands *L*1 (1,223-1,236 MHz) and *L*2 (1,575-1,590 MHz) (Fig. 7b).

The metamaterial sample effective permittivity and permeability value determination was made according to the following algorithm:

(1) Calibration of a vector network analyzer with connected cables.

(2) Connecting of the measuring tool and its complex transmission and reflection coefficients measured values obtaining.

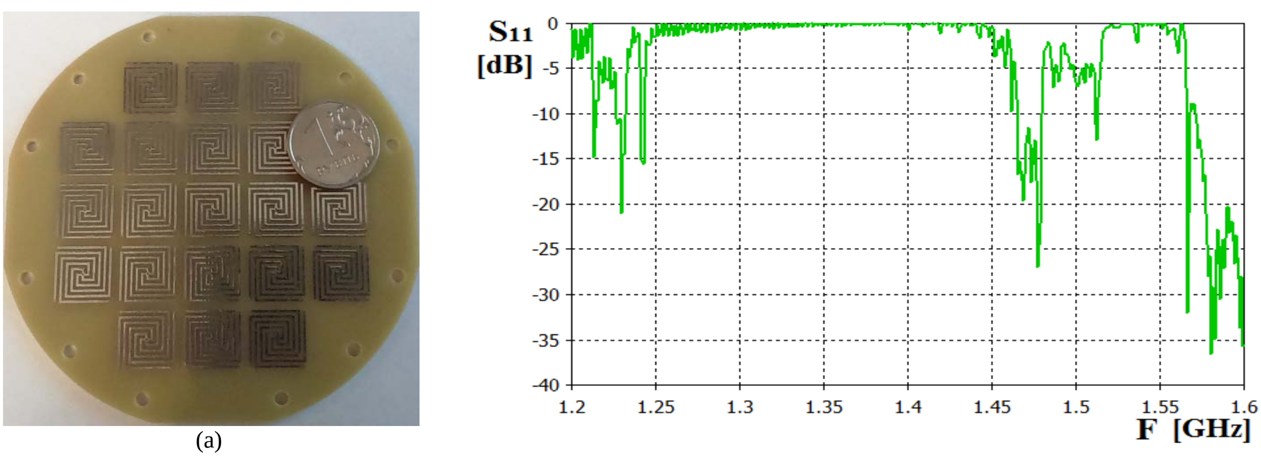
(3) Installing the sample under test into the tool and its complex transmission and reflection coefficients measured values obtaining.

(4) Calculation according to Eqs. (8) and (9) of the self (true) values of the complex transmission and reflection coefficients of the sample under test.

(5) Calculation of the permittivity and permeability effective values according to Eqs. (4)-(7) for each measuring point.

Figs. 8a-8d present the results of the complex reflection and transmission coefficient measurements according to the points 1-3 of the algorithm, made with the help of measuring tools, intended for cases of propagating along the metamaterial working surface of the electromagnetic wave with vertical and horizontal polarization of electric field. It is seen that in both cases the measured structure band gap positions correspond well with the result of simulation.

Figs. 9a-9d present the calculated values of the sample effective permittivity and permeability, obtained separately for cases of vertical and horizontal field polarization. Both calculations show that in the band gap frequency area the sample effective permittivity and permeability values are negative, what is a metamaterial unconditional property.



(a) (b)

Fig. 7 Metamaterial sample under test (a) and the result of its transmission coefficient simulation (b).

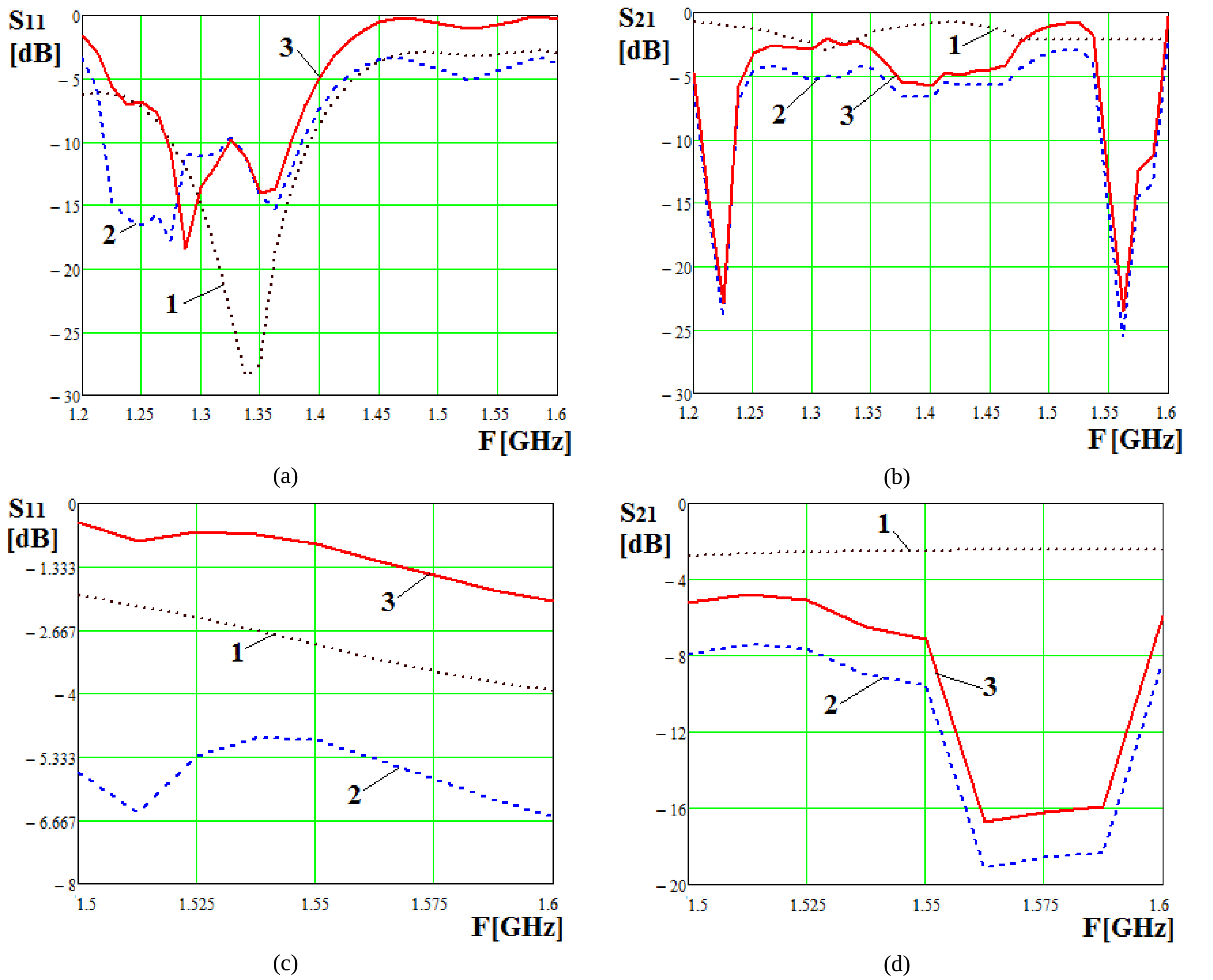
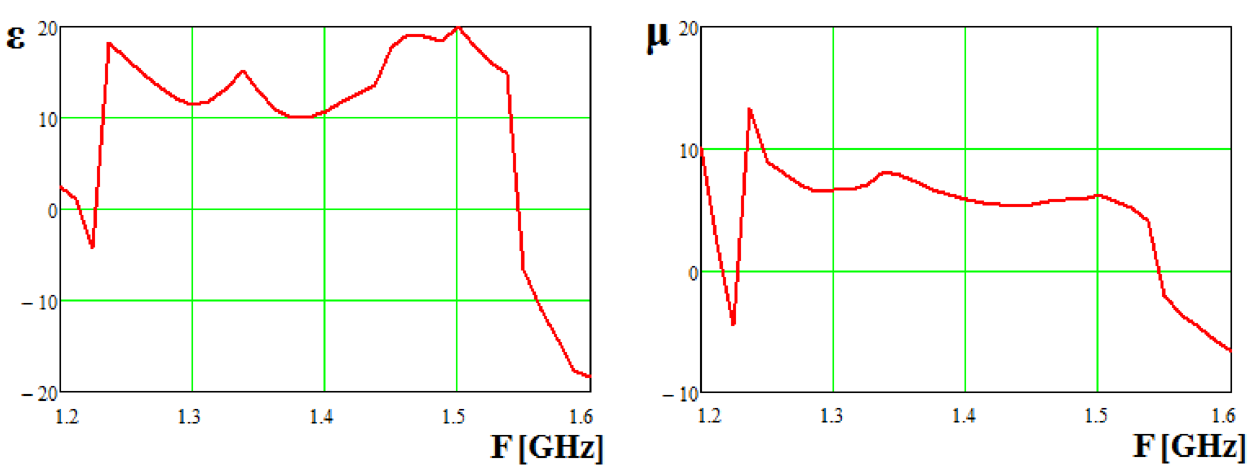
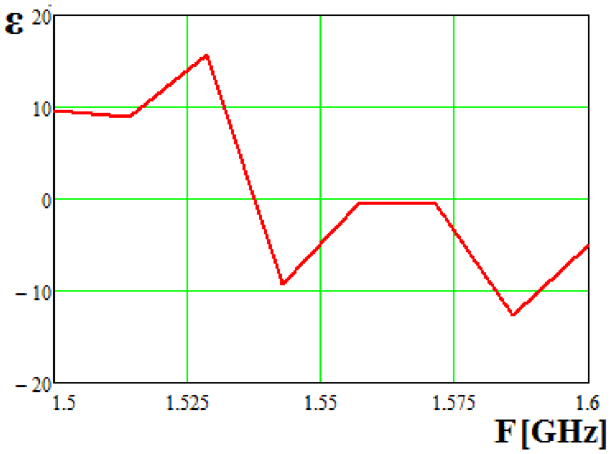


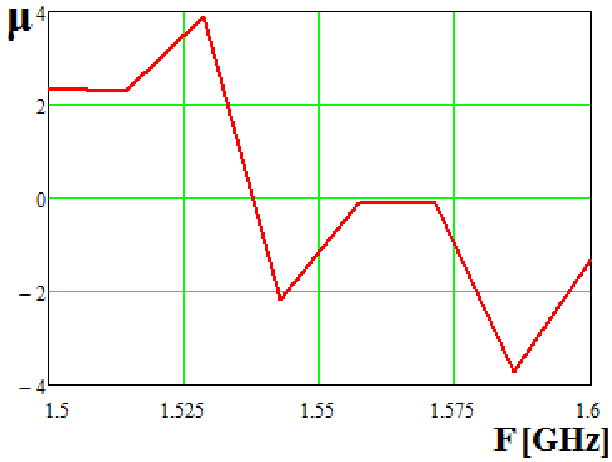
Fig. 8 **The measured values of complex transmission and reflection coefficients of the measuring tools (1) and the sample under test (2), and calculated self values of complex coefficients (3): (a) reflection coefficient in case of using the wave with vertical polarization; (b) transmission coefficient in case of using the wave with vertical polarization; (c) reflection coefficient in case of using the wave with horizontal polarization; (d) transmission coefficient in case of using the wave with horizontal polarization.**



(a) (b)



(c)



(d)

Fig. 9 Calculated values of measured metamaterial sample effective permittivity and permeability: (a) permittivity in case of vertically-polarized wave propagation along the metamaterial surface; (b) permeability in case of vertically-polarized wave propagation along the metamaterial surface; (c) permittivity in case of horizontally-polarized wave propagation along the metamaterial surface; (d) permeability in case of horizontally-polarized wave propagation along the metamaterial surface.

4. Conclusions

The paper presents the methodology of metamaterial effective permittivity and permeability values determination, which is based on measuring of complex reflection and transmission coefficients of the structures. The mathematical equations to calculate the parameters are provided. Several measuring tool constructions, which are necessary for parameter determination, and which can be used during the metamaterial sample investigations in case of different field polarizations, are shown. A practical measurement of S-parameters of a dual frequency mushroom-type metamaterial is made and its effective permittivity and permeability values are determined. The obtained experimental results have good correlation with the result of simulation.

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